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ECON 405
ECONOMIC GROWTH AND DEVELOPMENT **7 June 2014**
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Final
Ramsey Model

1. (**50 points**) Suppose the economy is characterized by a production function in the form $Y_t = K_t^{\alpha} L_t^{1-\alpha}$, where $L_t = L_0 e^{nt}$, $L_0 = 1$, and $n > 0$ and an overall utility function $U(c) = \int_0^{\infty} u(c_t) e^{-(\rho - n)t} dt$, where the instantaneous utility function *u(·)* belongs to the constant elasticity of intertemporal substitution (CIES) class: $u(c_t) = \frac{c^{1-\theta} - 1}{1-\theta}$, $\theta > 0$.

a. Solve the household's intertemporal utility maximization problem. (**10 points**)
b. Solve the firm's profit maximization problem. (**10 points**)
c. Solve the model at the steady state and find the equilibrium values of capital, output, and consumption. (**10 points**)
d. Formulate the same problem by using the social planner's approach. (**10 points**)
e. Suppose now that you are given the following parameter values: $\alpha = 0.5$, $\theta = 2$, $\rho = 0.1$, $n = 0.02$, $L_0 = 1$, $\delta = 0.05$. Calculate the steady state values of capital, output, and consumption. (**10 points**)

2. (**50 points**) Suppose that a social planner has the following optimization problem:

$$
\begin{aligned}
&U = \int_0^{\infty} e^{-\rho t} \frac{C^{1-\theta} - 1}{1-\theta} dt \\
&Y = ((1-u)H)^{\alpha} K^{1-\alpha} \\
&\dot{K} = Y - C \\
&\dot{H} = \delta u H
\end{aligned}
\qquad \text{(M)}
$$

C, Y and K represent consumption, output and the capital stock, respectively. ρ is the subjective rate of discount, while $1/\theta$ is the intertemporal elasticity of substitution for consumption. We assume that $\rho > 0$, $0 < \theta$, $0 < \alpha < 1$ where α is the partial output elasticity of labor. $1-u$ and u are the fractions of the skilled labor, H, allocated to the generation of output and skills, respectively. The production function is Cobb-Douglas. The macroeconomic budget shows the tradeoff between consumption and investment. The last equation is the human capital accumulation function, which generates more skilled-labor, and δ is the productivity of the human capital generation process. Solve the model.